INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

SENIOR PAPER: YEARS 11,12

Tournament 42, Northern Autumn 2020 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Each of the quadratic polynomials $P(x)$, $Q(x)$ and $P(x) + Q(x)$ with real coefficients has a root of multiplicity 2. Is it always the case that the roots are the same? (3 points)

2. There are 10 points X_1, X_2, $\ldots$, X_{10} on a line in that order. Each of the line segments X_1X_2, X_2X_3, $\ldots$, X_9X_{10} is the base of an isosceles triangle with the angle α opposite each such base. Suppose that all vertices opposite the corresponding bases of the triangles constructed lie on a common semicircle with diameter X_1X_{10}. Find α. (4 points)

3. A positive integer N is a multiple of 2020. All digits of N are different and if any two of them are swapped, the new number is not a multiple of 2020. How many digits can N have? (All numbers are given in base 10). (5 points)

4. The sides of a triangle are divided by the angle bisectors into two parts each. Is it always possible to form two triangles from the 6 line segments obtained in this way? (5 points)

5. There are 101 coins placed on a circle, each coin weighs 10 or 11 g. Prove that there exists a coin such that the total weight of the k coins to the left of that coin is equal to the total weight of the k coins to the right of that coin if

   (a) $k = 50$. (3 points)

   (b) $k = 49$. (3 points)